Mobile Communications

TCS 455

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Lecture 19

Office Hours:
BKD 3601-7
Tuesday 14:00-16:00
Thursday 9:30-11:30

Announcements

- Read
  - Chapter 9: 9.1 – 9.5
- No class on Jan 21 (Next Thursday) – university game
- HW5 will be posted next week.
  - Due: After university game

Chapter 4

Multiple Access

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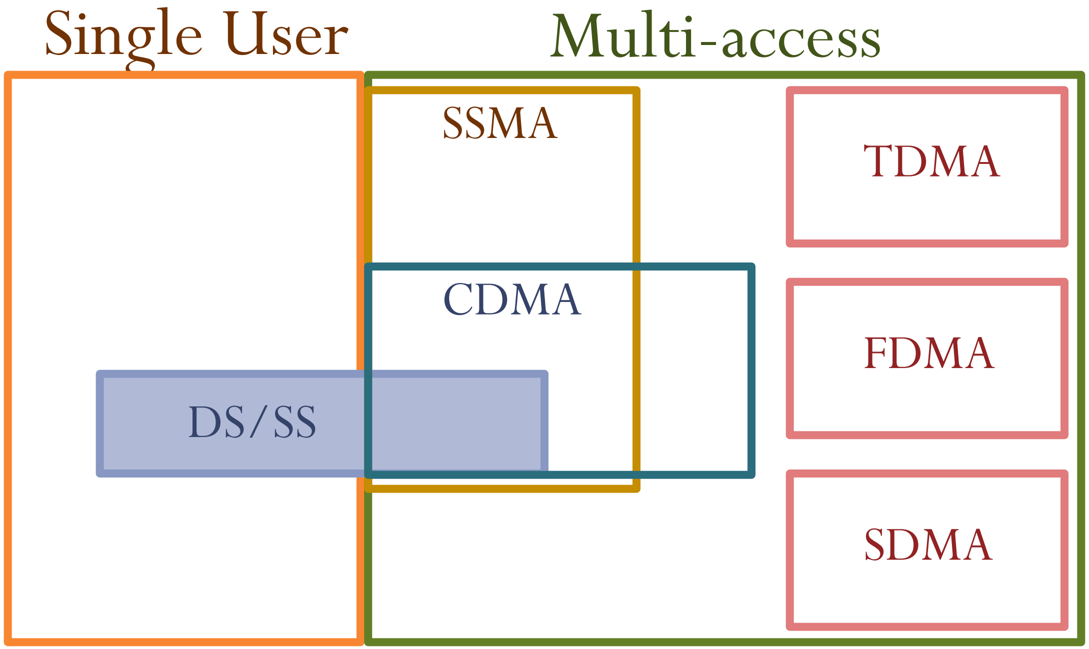
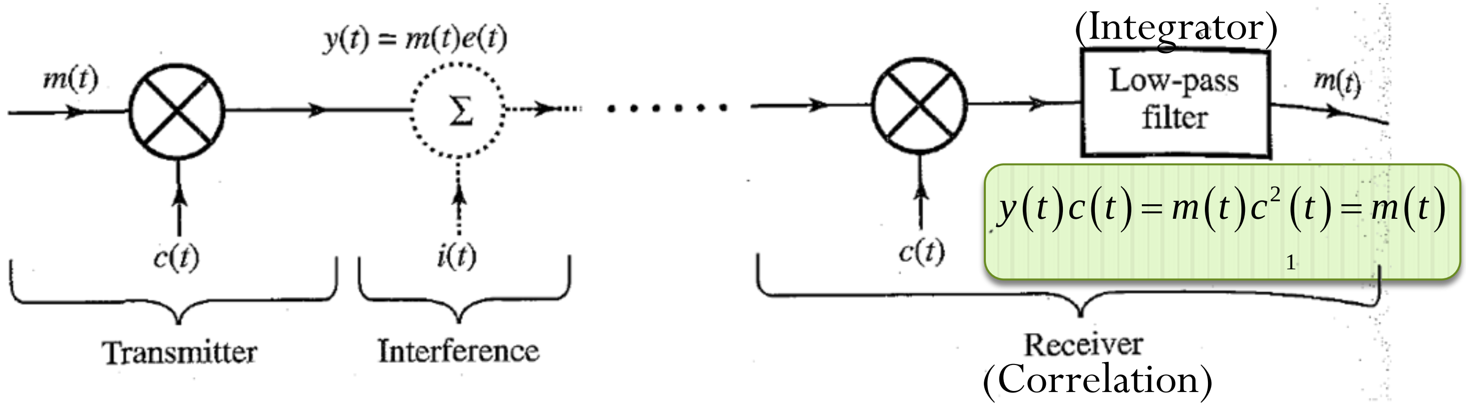
SSMA, CDMA, DS/SS



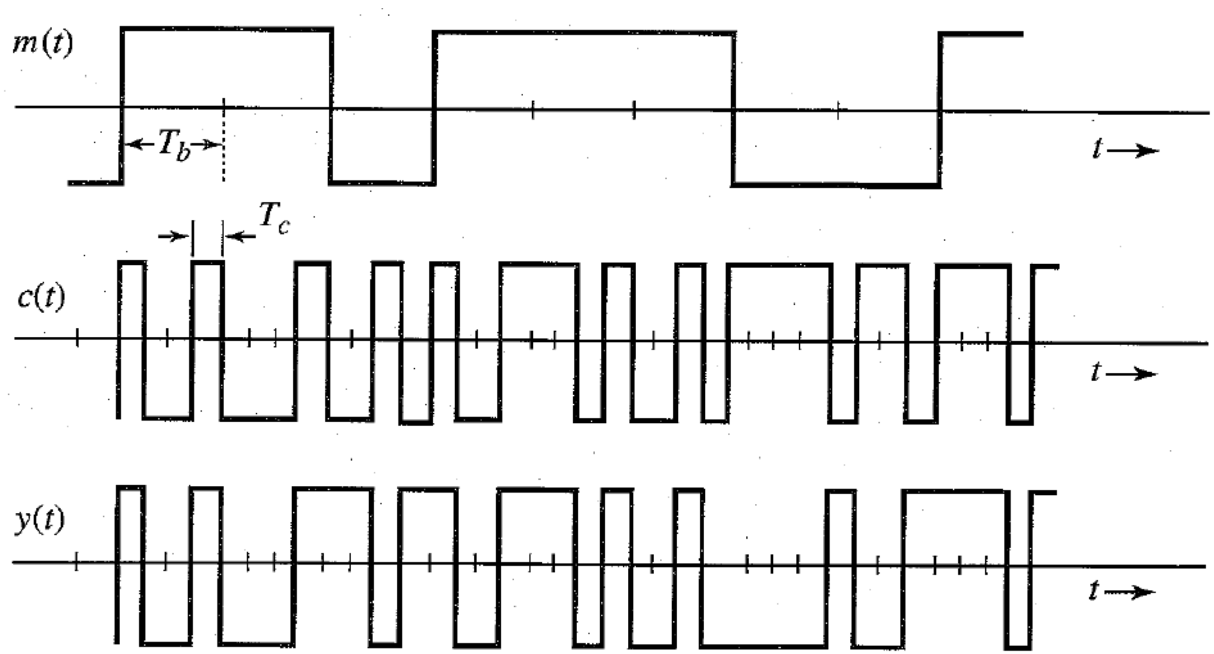
Useful even for single user!

DS/SS System

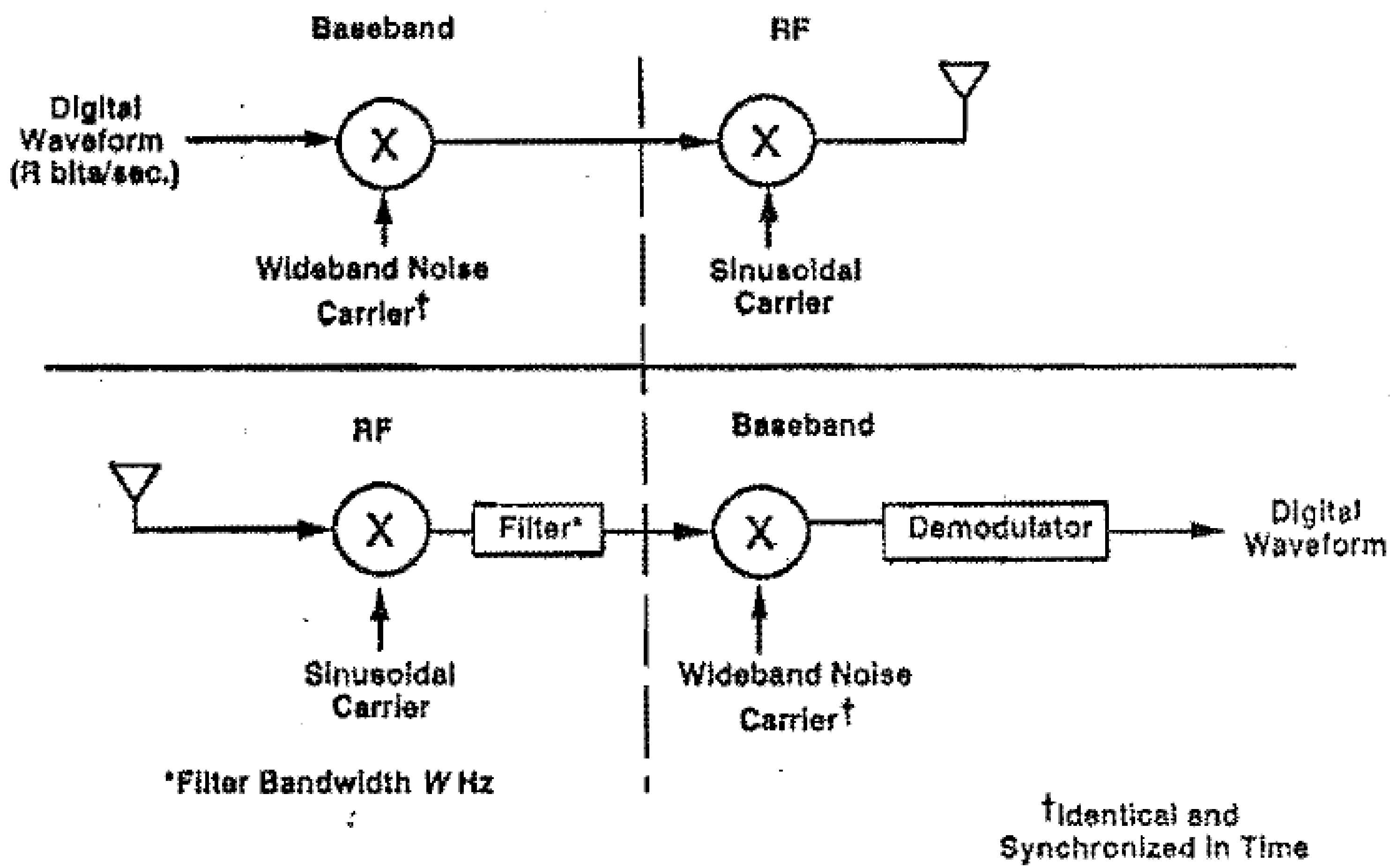


Message signal (polar binary signal)

Polar signal representing **psuedonoise (PN)** sequence. (Think of this as a pseudorandom carrier)



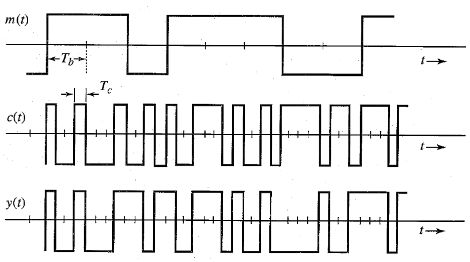
Spread spectrum modem



[Viterbi, 1995, Fig. 1.2]

DS/SS

- The spectral spreading signal c(t) is a pseudorandom signal
  - Appear to be unpredictable
  - Can be generated by deterministic means (hence, pseudorandom)
- The bit rate of c(t) is chosen to be much higher then the bit rate of m(t).
- The basic pulse in c(t) is called the **chip**.
- The bit rate of c(t) is known as the **chip rate**.
- The auto correlation function of c(t) is very narrow.
  - Small similarity with its delayed version



- Remark: In multiuser (CDMA) setting, the cross-correlation between any two codes $c_1(t)$ and $c_2(t)$ is very small
  - Negligible interference between various multiplexed signals.
- Notice that the process of detection (despreading) is identical to the process of spectral spreading.
  - Recall that for DSB-SC, we have a similar situation in that the modulation and demodulation processes are identical (except for the output filter).

Binary Random Sequences

- While DSSS chip sequences must be generated *deterministically*, properties of binary random sequences are useful to gain insight into deterministic sequence design.
- A random binary chip sequences consists of i.i.d. bit values with probability one half for a one or a zero.
  - Also known as Bernoulli sequences, "coin-flipping" sequences
- A random sequence of length N can be generated, for example, by flipping a fair coin N times as setting the bit to a one for heads and a zero for tails.

Pseudorandom Sequence

- **Key randomness properties** [Golomb, 1967]]: Binary random sequences with length *N* asymptotically large have a number of the properties desired in spreading codes
  - **Balanced property** of a code: Equal number of ones and zeros.
  - **Run length property** of a code: The run length is generally short.
    - half of all runs are of length 1
    - a fraction $1/2^n$ of all runs of length *n* (Geometric)
  - **Shift property** of a code: If they are shifted by any nonzero number of elements, the resulting sequence will have half its elements the same as in the original sequence, and half its elements different from the original sequence.
- A deterministic sequence that has the balanced, run length, and shift properties as it grows *asymptotically large* is referred to as a **pseudorandom sequence** (noiselike signal).

Note: A run is a subsequence of identical symbols within the sequence.

Pseudonoise (PN) signature sequence

- Ideally, one would prefer a random binary sequence as the spreading sequence.
- However, practical synchronization requirements in the receiver force one to use periodic Pseudorandom binary sequences.
- m-sequences
- Gold codes
- Kasami sequences
- Quaternary sequences
- Walsh functions

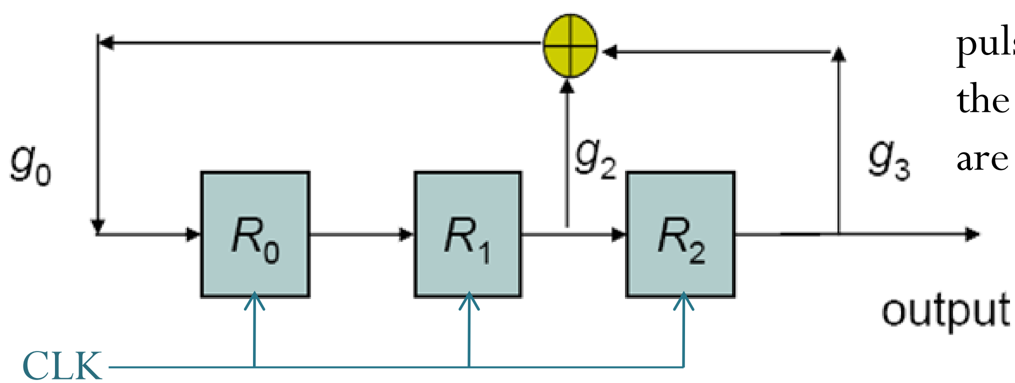
m-Sequences (1)

- **Maximal-length sequences**
- A type of **cyclic code**
  - Generated and characterized by a generator polynomial
  - Properties can be derived using algebraic coding theory
- Simple to generate with **linear feedback shift-register** (LFSR) circuits
  - Automated
- Approximate a random binary sequence in the sense that shifted versions of itself are approximately uncorrelated.
- Relatively easy to intercept and regenerate by an unintended receiver

m-sequence generator

- The feedback taps in the feedback shift register are selected to correspond to the coefficients of a **primitive polynomial**.

Binary sequences drawn from the alphabet {0,1} are shifted through the shift register in response to clock pulses. The particular 1s and 0s occupying the shift register stages after a clock pulse are called **states**.



$g(x) = x^3 + x^2 + 1$ (Degree: r = 3)

The g_i's are coefficients of a primitive polynomial.

1 signifies closed or a connection and
0 signifies open or no connection.

Time	R_0	R_1	R_2
0	1	0	0
1	0	1	0
2	1	0	1
3	1	1	0
4	1	1	1
5	0	1	1
6	0	0	1
7	1	0	0

Sequence repeats from here onwards

GF(2)

- **Galois field** (finite field) of two elements
- Consist of
  - the symbols 0 and 1 and
  - the (binary) operations of
    - **modulo-2** addition (XOR) and
    - **modulo-2** multiplication.
- The operations are defined by

$$0 \oplus 0 = 0, \quad 0 \oplus 1 = 1, \quad 1 \oplus 0 = 1, \quad 1 \oplus 1 = 0$$

$$0 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 1 \cdot 0 = 0, \quad 1 \cdot 1 = 1$$

m-Sequences: More properties

- The contents of the shift register will cycle over all possible 2^r-1 nonzero states before repeating.
- Contain one more 1 than 0
- Sum of two **(cyclic-)shifted** m-sequences is another (cyclic-)shift of the same m-sequence
- If a window of width *r* is slid along an m-sequence for $N = 2^r - 1$ shifts, each *r*-tuple except the all-zeros r-tuple will appear exactly once
- For any m-sequence, there are
  - One run of ones of length *r*
  - One run of zeros of length *r*-1
  - One run of ones and one run of zeroes of length r-2
  - Two runs of ones and two runs of zeros of length r-3
  - Four runs of ones and four runs of zeros of length r-4
  - …
  - 2^{r-3} runs of ones and 2^{r-3} runs of zeros of length 1

[S.W. Golomb, *Shift Register Sequences,* Holden-Day, San Francisco, 1967.]

Ex: Properties of m-sequence

00101110010111001011100101110010111001011100101110010111

Runs:

111

00

1,0

0 phase shift: 0010111

1 phase shift: 0101110

2 phase shift: 1011100

3 phase shift: 0111001

4 phase shift: 1110010

5 phase shift: 1100101

6 phase shift: 1001011

$\oplus$ = 1100101

00101110010111001011100101110010111001011100101110010111

Ex: Properties of m-sequence (con't)

- 2^5-1 = 31-chip m-sequence

1010111011000111110011010010000

1010111011000111110011010010000

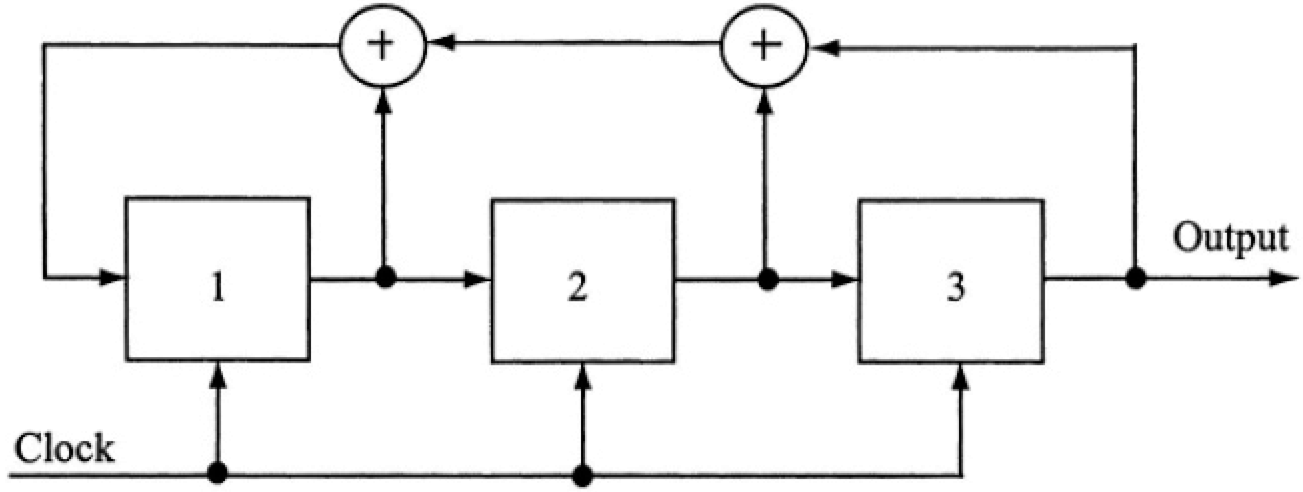
Runs:

11111	1
0000	1
111	1
000	1
11	2
00	2
1	4
0	4

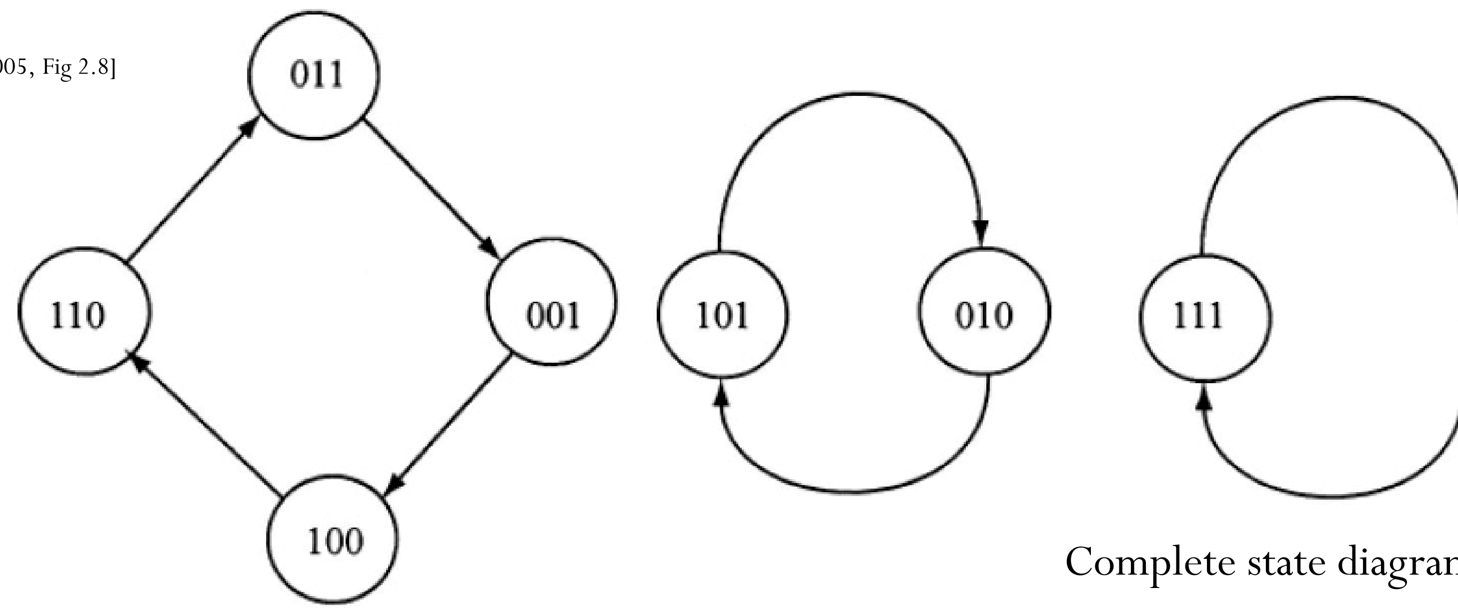
There are 16 runs.

Nonmaximal linear feedback shift register

$$x^3 + x^2 + x + 1$$



[Torrieri , 2005, Fig 2.8]



Complete state diagrams

m-Sequences (con't)

0010111001011100101110010111001011100101110010111001011100101110010111

0010111

$\downarrow$

$\oplus$

$\downarrow$

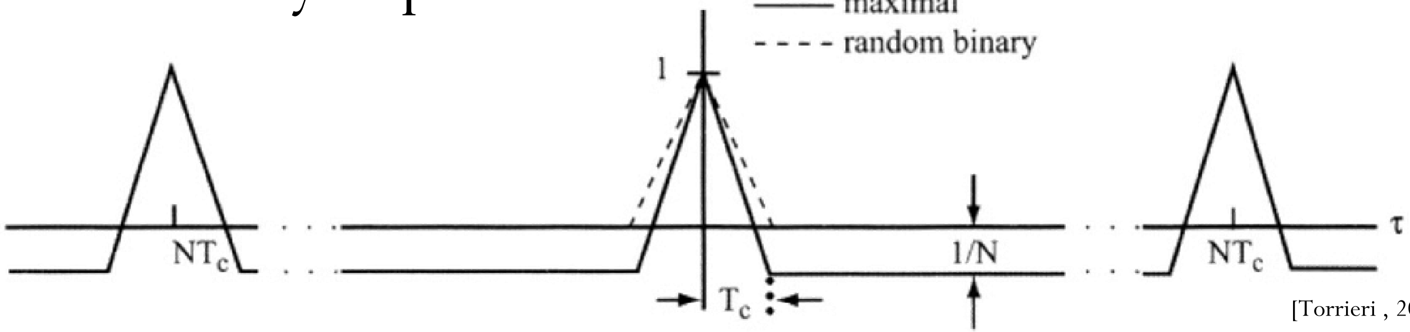
1001011

In actual transmission, we will map 0 and 1 to +1 and -1.

-1	1	-1	-1	-1	1	1	×
1	1	-1	1	-1	-1	-1	
-1	1	1	-1	1	-1	-1	$\sum = -1$

Autocorrelation and PSD

- (Normalized) autocorrelations of maximal sequence and random binary sequence.



[Torrieri , 2005, Fig 2.9]

- Power spectral density of maximal sequence.

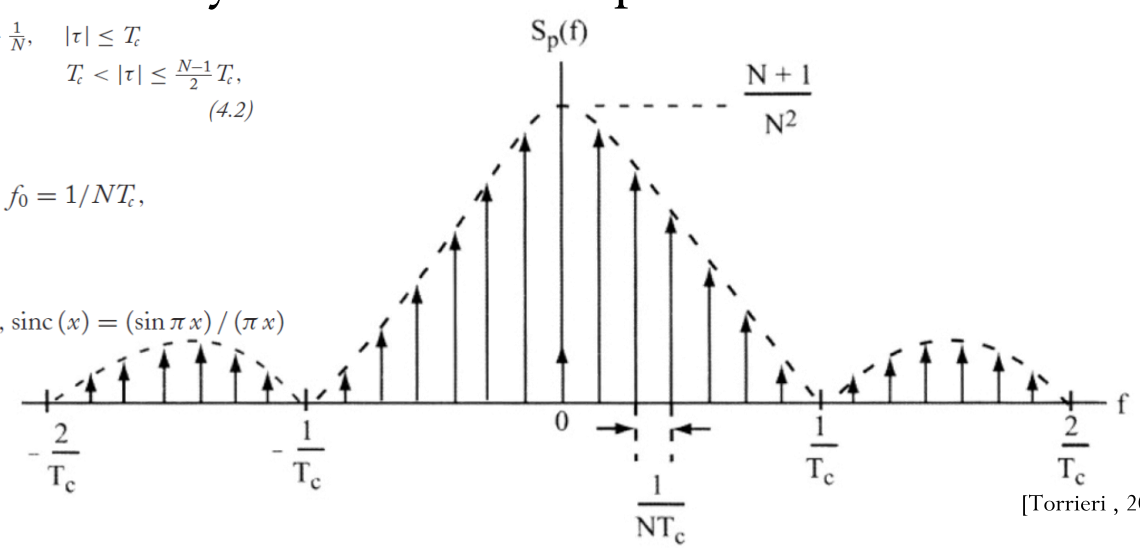
$$R_c(\tau) = \frac{1}{T_0}\int_{T_0} x(t)x(t+\tau)\,dt = \begin{cases} \left(1-\frac{|\tau|}{T_c}\right)\left(1+\frac{1}{N}\right)-\frac{1}{N}, & |\tau| \le T_c \\ -\frac{1}{N}, & T_c < |\tau| \le \frac{N-1}{2}T_c, \end{cases} \quad (4.2)$$

where the integration is over any period, $T_0 = NT_c$.

$$S_c(f) = \sum_{m=-\infty}^{\infty} P_m \delta(f - mf_0), \; f_0 = 1/NT_c,$$

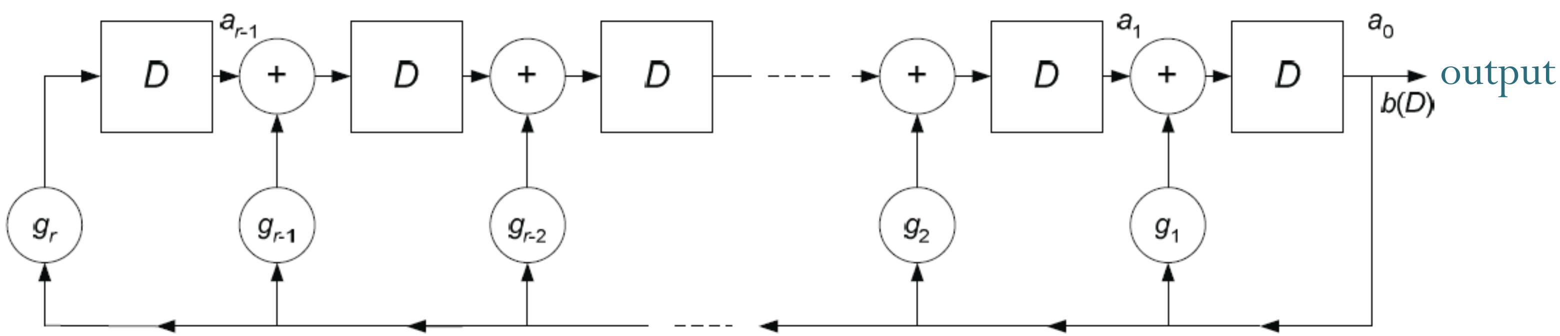
where

$$P_m = \begin{cases} \left[(N+1)/N^2\right] \text{sinc}^2(m/N), \; m \ne 0, \; \text{sinc}(x) = (\sin \pi x)/(\pi x) \\ 1/N^2, \; m = 0. \end{cases}$$

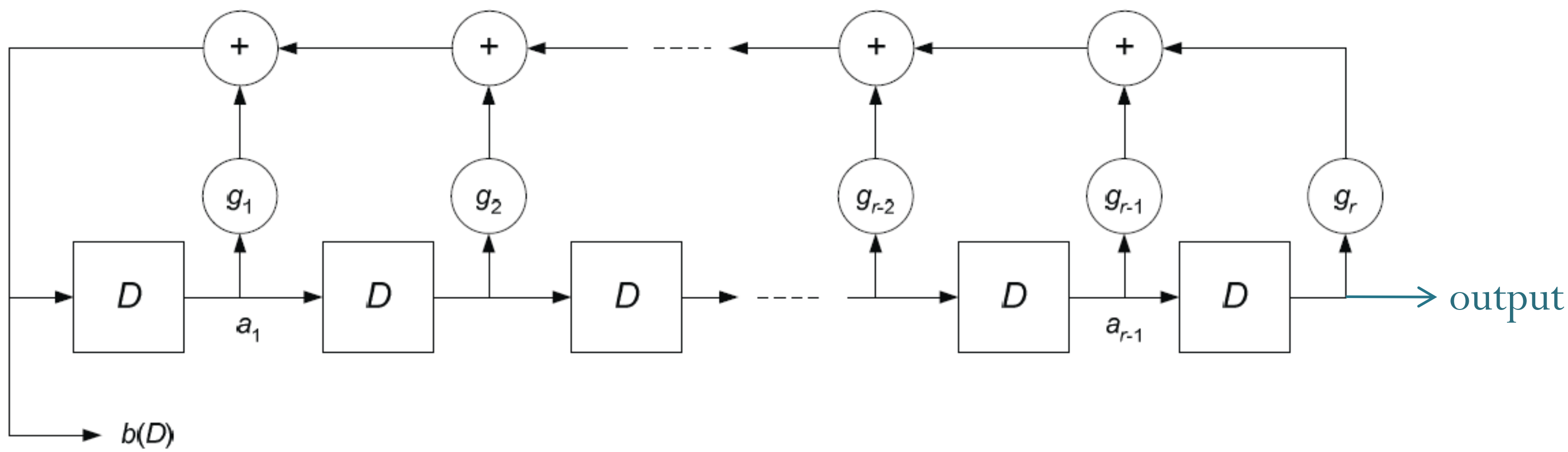


[Torrieri , 2005, Fig 2.10]

Two configurations of m-sequence generators



(a) High-speed linear feedback shift-register generator



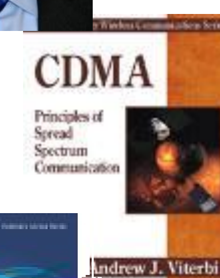
(b) Low-speed linear feedback shift-register generator (standard form)

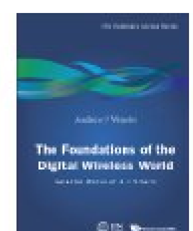
[Ziemer, 2007, Fig. 5]
[Torrieri,2005, Fig. 2.7]

Code Division Multiple Access (CDMA)

- Qualcomm
- Founders: two of the most eminent engineers in the world of mobile radio
  - Irwin Jacobs is the chairman and founder
  - Andrew J. Viterbi is the co-founder
    - Same person that invented the Viterbi algorithm for decoding convolutionally encoded data.
- 1991: Qualcomm announced
  - that it had invented a new cellular system based on CDMA
  - that the capacity of this system was 20 or so times greater than any other cellular system in existence
- However, not all of the world was particularly pleased by this apparent breakthrough—in particular, GSM manufacturers became concerned that they would start to lose market share to this new system.
  - The result was continual and vociferous argument between Qualcomm and the GSM manufacturers.





CDMA

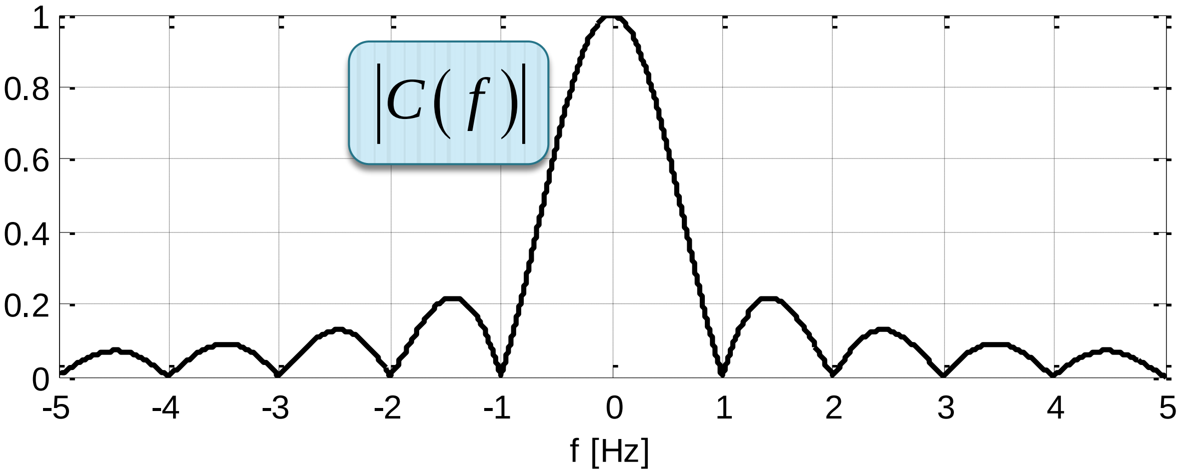
- One way to achieve SSMA
- May utilize Direct Sequence Spread Spectrum (DS/SS)
  - Direct sequence is not the only spread-spectrum signaling format suitable for CDMA
- All users use the same carrier frequency and may transmit simultaneously.
- Users are assigned different "**signature waveforms**" or "code" or "codeword" or "**spreading signal**"

Not to be confused with error-correcting codes that add redundancy to combat channel noise and distortion

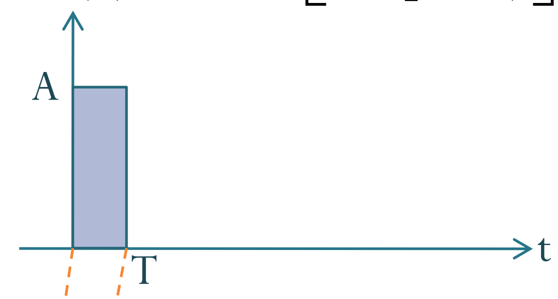
- The narrowband message signal is multiplied (modulated) by the **spreading signal** which has a very large bandwidth (orders of magnitudes greater than the data rate of the message).
- Each user's codeword is ***approximately orthogonal*** to all other codewords.
- Should not be confused with the mobile phone standards called cdmaOne (Qualcomm's IS-95) and CDMA2000 (Qualcomm's IS-2000) (which are often referred to as simply "CDMA")
  - These standards use CDMA as an underlying channel access method.

Recall: TDMA Spectrum

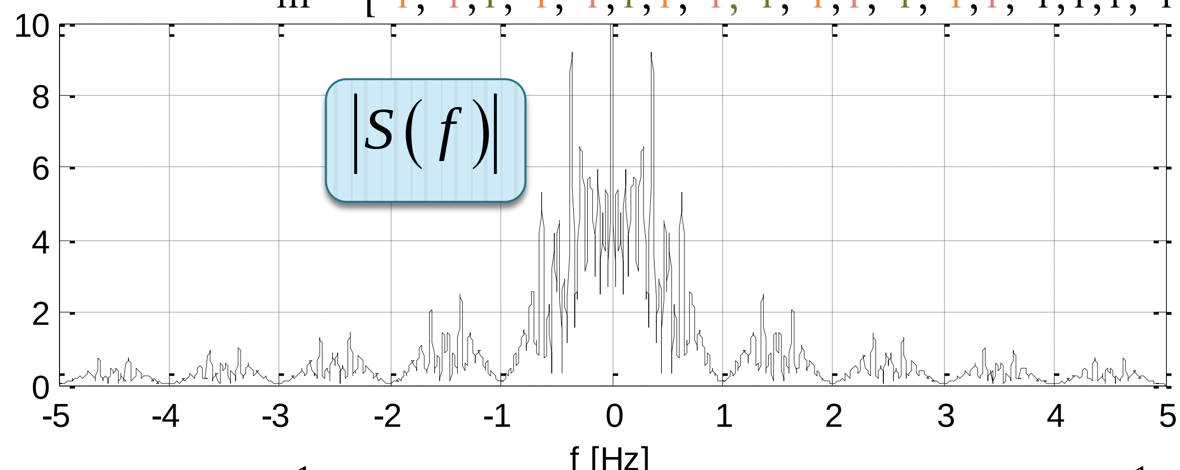
$(A=1, T=1)$

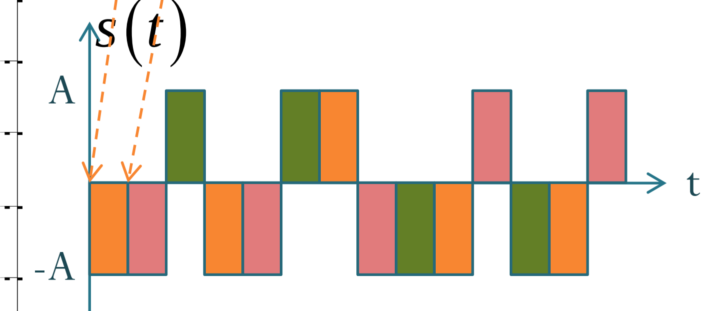


$$c(t) = A \times 1\left[t \in [0,T)\right]$$



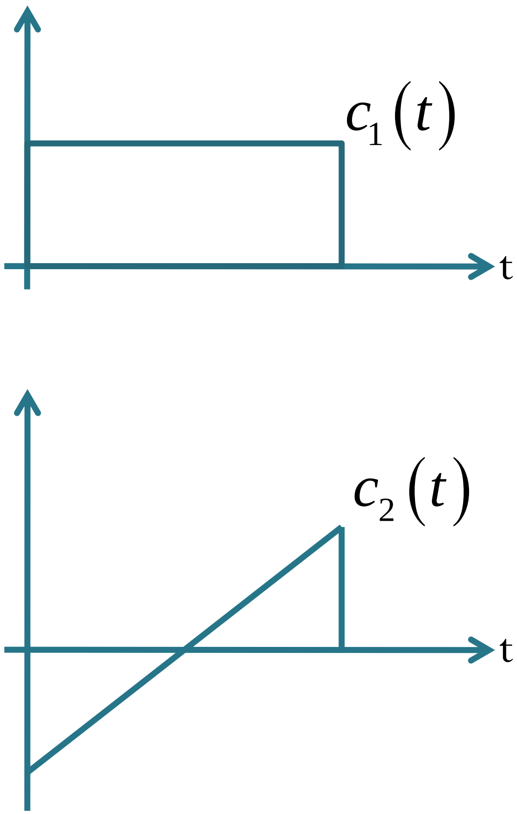
m = [-1,-1,1,-1,-1,1,1,-1,-1,-1,1,-1,-1,1,-1,1,1,-1,-1,-1,-1,1,-1,-1,-1,-1,-1,1,-1,1]

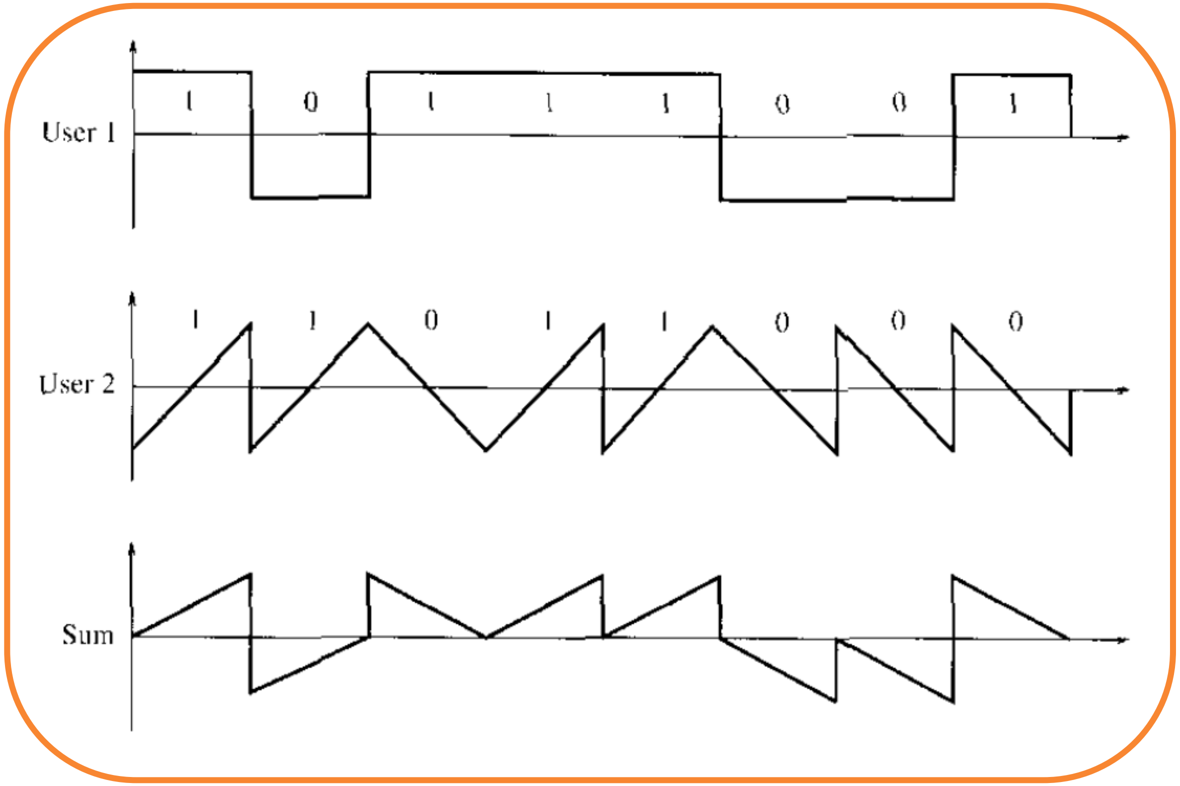




$$s(t) = \sum_{k=0}^{n-1} m_k c(t-kT) \xrightarrow{\mathcal{F}} S(f) = C(f) \sum_{k=0}^{n-1} m_k e^{-j2\pi fkT}$$

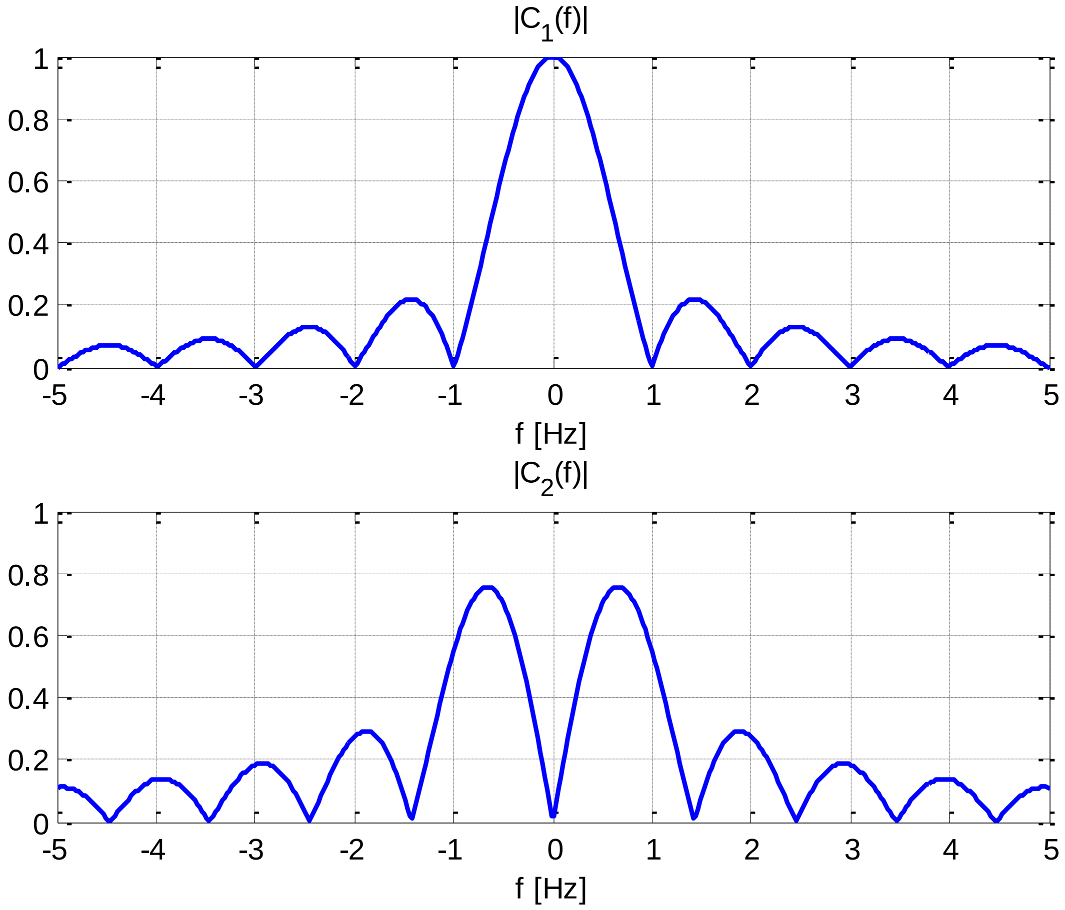
CDMA: Simplified Example (1)





[Figure 1.6, Verdu, 1998]

CDMA: Simplified Example (1) – con’t



CDMA

- Reception free from inter-channel interference is a consequence of the use of orthogonal signaling.
- This can be accomplished by signals that overlap both in time and in frequency.
- **Orthogonal (real-valued) signals**:

Inner product, Cross-correlation →

$$\langle c_1, c_2 \rangle = \int_0^T c_1(t)\, c_2(t)\, dt = 0$$

- Special case: TDMA
  - The signature waveforms do not overlap in the time domain.